

Local Time and the Unification of Physics

Part I. Local Time

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Abstract: The notions of time in the theories of Newton and Einstein are reviewed so that the difficulty which impedes the unification of quantum mechanics (QM) and general relativity (GR) is clarified. It is seen that GR by itself contains an intrinsic difficulty relating to the definition of local clocks, as well as that GR still requires a kind of absolute that can serve as an objective reference standard. We present a new understanding of time, which gives a consistent definition of a local time associated with each local system in a quantum mechanical way, so that it serves the requirements of both GR as well as QM. As a consequence, QM and GR are reconciled while preserving the current mathematical formulations of both theories.

I. Introduction

Previous papers of Kitada 1994a, 1994b proposed an approach to the problem of overcoming the apparent inconsistency of non-relativistic quantum mechanics and general relativity. The purpose of this paper is to explain the structure and background of that approach, with emphasis on a certain philosophical problem related with the notion of time.

The inconsistency of quantum mechanics and general relativity, when looked at mathematically, seems at first sight obvious and inescapable from the fact that the geometry of quantum mechanics is Euclidean, while general relativity employs a curved, Riemannian geometry.

Kitada 1994a proposes to overcome this apparent mathematical incommensurability of these two geometries by “orthogonalizing” them; *i.e.* by expressing them as a direct product $X \times R^6$, where X represents the curved Riemannian manifold associated with general relativity, and R^6 (or in the usual space-time context, R^4) denotes the Euclidean space of phase space coordinates (x, v) of non-relativistic quantum mechanics. As two components of the orthogonalized total space $X \times R^6$, the Riemannian space X and the Euclidean space R^6 are compatible without contradiction.

General relativity and quantum mechanics are the two most important and comprehensive theories of contemporary physics. By “comprehensive” we mean that both theories claim to apply to everything. In practice it might seem that these two theories describe two different physical domains, since the most striking applications of quantum mechanics occur when we consider things that are extremely tiny in relation to ourselves – things like electrons and photons – while the most striking applications of general relativity occur in connection with extremely large and dense concentrations of matter and enormous spatio-temporal magnitudes. But, in principle, every physical thing must be capable of being described adequately by both theories, at least this is what the theories claim. And there are certain cases – of particular interest in recent cosmology and astrophysics – where the extremes of density that are the particular province of general relativity coincide with the extremes of minuteness that are the special province of quantum mechanics. In those situations, the physicist is compelled to face a problem which is present in the background of science all the time but which can otherwise be evaded without practical consequence: the fact, namely, that these two comprehensive theoretical structures appear to be mutually incompatible, that they seem to involve different – and contradictory – assumptions about the nature of space, time and causality.

Our intention in this paper is to outline an approach to the understanding of general relativity and quantum mechanics in which these theories will appear as distinct but systematically coordinated perspectives on the same reality. The orthogonalization of the spatial foundations of the two theories allows us to speak of the two theories as distinct. To express the possibility of their systematic coordination will require a more extended analysis of the nature of time.

In brief, we can express our approach as follows:

1. We begin by distinguishing the notion of a local system consisting of a finite number of particles. Here we mean by “local” that the positions of all particles in a local system are understood as defined with respect to the same reference frame.
2. In so far as the particles comprised in this local system are understood locally, we note that these particles are describable only in terms of quantum mechanics. In other words, to the extent that we consider the particles solely within the local reference frame, these particles have only quantum mechanical properties, and cannot be described as classical particles in accordance with general relativity.
3. Next we consider the center of mass of a local system. Although the local system is considered as composed of particles which – as local – have only quantum mechanical properties, in our orthogonal approach we posit that each point (t, x) in the

Riemannian manifold X is correlated to the center of mass of some local system. Therefore, in our approach, the classical particles whose behavior is described by the general theory of relativity are *not* understood as identical with the “quantum mechanical” particles inhabiting the local system – rather the classical particles are understood as precisely correlated only with the centers of mass of the local systems.

4. It is important to recognize that the distinction we are making between local systems and classical particles which are the centers of mass of local systems is not a simple distinction of inclusion/exclusion. For example, we may consider a local system containing some set of particles, and within that set of particles we may identify a number of subordinate “sublocal” systems. It would seem that the centers of mass of these sublocal systems must be “inside” the local system as originally defined, but the sublocal system is at the same time a local system, and we have said that the centers of mass of local systems are correlated with classical particles whose behavior is to be described in terms of relativity theory.

The paradox is avoided by noting that the distinction we are making is a distinction of reference frame, not a distinction of inclusion or exclusion. When we speak of classical particles (or centers of mass) we are speaking of the particle in terms of the observer’s time, which is understood as distinct from that of the particle observed. To the extent that the time of the system L itself is adopted as the reference time, then we are speaking of the behavior of a local system whose development must be described in terms of quantum mechanics¹.

It is our contention that time necessarily has two quite different aspects, in relativity theory, on the one hand, and in quantum theory on the other, and the intention of this paper is to show that these two aspects of time are in fact complementary and that the notion of local time, which we have associated with the quantum mechanical local system, is not only the main ingredient of a unification of quantum and relativity theories, but that this actually is necessary to constituting the time of relativity theory.

The “orthogonalization” of the geometries of quantum mechanics and general relativity does not by itself specify the nature of the relationship between them. It simply gives us a way of representing them as independent but complementary. The nature of that relationship, and the value of this form of representation, will come to light in Part II, after we outline our notion of local time below.

Before stating our notion of local time, it will be useful to show how this notion relates to the apparent inconsistency of quantum mechanics and general relativity.

II. Time in Quantum Mechanics

That the difficulty of reconciling quantum mechanics and general relativity is connected to the question of time is now generally recognized (see especially Isham 1993, Unruh 1993

¹The mathematical details of the relationship between the local time and the observer’s time will be set forth in Part II, after we have developed our notion of local time in section IV of this part.

and Hartle 1993). What is central is the divergent relationship of these two branches of modern physics to their common Newtonian heritage.

At the beginning of modern physics, Isaac Newton specified his notion of time in the *Principia* as follows (Newton 1962, p.6):

Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.

Also in pp.7-8, he states:

Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the apparent time. For the natural days are truly unequal, though they are commonly considered as equal, and used for a measure of time; astronomers correct this inequality that they may measure the celestial motions by a more accurate time. It may be, that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any change. The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore this duration ought to be distinguished from what are only sensible measures thereof; and from which we deduce it, by means of the astronomical equation. The necessity of this equation, for determining the times of a phenomenon, is evinced as well from the experiments of the pendulum clock, as by eclipses of the satellites of Jupiter.

The main point of this famous passage is to assert the existence of an absolute, true time. However, it is important to note that Newton asserts the existence of his absolute time by means of a distinction. There is absolute time, which flows without reference to anything external, and then there is relative, apparent, or common time, which is a measure of duration made by comparison of motions. Not only that, but although there may be no absolutely regular motion by means of which absolute time may be accurately represented, absolute time is an ideal standard by means of which relative or common time is “corrected.”

Einstein’s theory of relativity, as is well-known, sharply contrasts with Newton precisely on the question of time and space: Einstein’s theory makes no reference to either absolute time or absolute space. Einstein retains the relative or common time which can be measured and determined by means of actual clocks associated with each local observer, but he completely jettisons Newton’s notion of an absolute time flowing equably for all observers.

But precisely in this respect, quantum mechanics stands in sharp contrast to relativity – especially the general theory of relativity. In quantum mechanics, unlike relativity, the time parameter continues to be treated in an essentially Newtonian manner.

That time plays a special, absolute role in quantum mechanics is evident in Schrödinger equation (at least as customarily interpreted):

$$\frac{\hbar}{i} \frac{\partial}{\partial t} \psi(x, t) + H\psi(x, t) = 0, \quad \psi(x, 0) = \psi_0(x),$$

where the Schrödinger operator or the Hamiltonian H of the system is defined by

$$H\psi(x, t) = -\frac{\hbar^2}{2m} \sum_{j=1}^3 \frac{\partial^2 \psi}{\partial x_j^2}(x, t) + V(x)\psi(x, t).$$

Thus the solution of the Schrödinger equation is given by

$$\psi(x, t) = \exp[-itH/\hbar]\psi_0.$$

In this context, the time t appears to be given *a priori*, and then the motion $\psi(x, t)$ of the system is derived from the Schrödinger equation by using the time evolution $\exp[-itH/\hbar]$ of the system².

The fact that time plays a special role in quantum mechanics may also be seen by looking at the alternative formulation of quantum mechanics by Feynman 1948. See also Kitada 1980 for the relation between the classical mechanics and quantum mechanics researched along the line given by Feynman 1948.

Because in non-relativistic quantum mechanics the time-evolution of a system is governed by the Schrödinger equation, space and time in quantum mechanics are intrinsically Newtonian in the sense that the form of Schrödinger equation is not invariant with respect to the relativistic transformation of coordinates.

III. What would an adequate notion of time require?

We have indicated that the primary source of the inconsistency between QM and GR is to be found in their divergent and apparently incompatible ways of treating time. Einstein pointedly rejects Newton's idea of absolute time and treats time as something which is locally defined by means of clocks which are at the same time physical objects. In non-relativistic QM, on the other hand, the role of time is essentially Newtonian, in the sense that time is an external, background parameter.

Newton's statements about absolute time and space were controversial from the time they were first published, and Einstein was by no means the first to call them into question. Einstein seems to have considered the Newtonian absolutes as purely metaphysical in nature, having no direct bearing on actual physical description – even for classical mechanics. In his own presentations Einstein is consistently operational. The description of spatial relations, prior to the introduction of GR, is explained as involving the specification of places on rigid reference bodies and spatial co-ordinate systems are understood as convenient, abstract mathematical substitutes for such rigid bodies of reference. Time

²See Kitada 1994a and sections IV and V below for a different interpretation of this solution, which will supply the key to our notion of a 'local clock'.

is understood, not as some absolute parameter, but as something based on the readings of identically constructed clocks held in the hands of different observers, who match up the “ticks” of their clocks with the observed positions of the various objects which they are observing.

In short, it looks as if Einstein considered the Newtonian absolutes completely superfluous and thought that they could be disposed of with no consequence, and in fact much gain in scientific rigor. However, once we inquire into the epistemological function of the Newtonian absolutes we may discover that it is not in fact quite so easy to get rid of them without any consequence.

Clearly, the epistemological function of the Newtonian absolutes is to serve as a common reference standard. The idea is that there must be something which is the same for all observers, in terms of which all can be described. To that extent, it is clear that in the physics of Einstein, Newton’s absolutes have not actually been removed – they have only been disguised. So the notions of a rigid body and a “standard clock” are essentially surrogates for absolute space and time. This can be seen to some extent by comparing Weyl’s definition of a clock with Newton’s definition of absolute time:

Weyl 1952 (p.7) has this to say on the question of clocks and the measurement of time:

To be able to apply mathematical conceptions to questions of Time we must postulate that it is theoretically possible to fix in Time, to any order of accuracy, an absolutely rigorous **now** (present) as a **point of Time** – i.e. to be able to indicate points of time, one of which will always be the earlier and the other the later. The following principle will hold for this “order-relation”. If A is earlier than B and B is earlier than C, then A is earlier than C. Each two points of Time, A and B, of which A is the earlier, mark off a **length of time**; this includes every point which is later than A and earlier than B. The fact that Time is a form of our stream of experience is expressed in the idea of **equality**: the empirical content which fills the length of Time AB can in itself be put into any other time without being in any way different from what it is. The length of time which it would then occupy is equal to the distance AB. This, with the help of the principle of causality, gives us the following objective criterion in physics for equal lengths of time. If an absolutely isolated physical system (i.e. one not subject to external influences) reverts once again to exactly the same state as that in which it was at some earlier instant, then the same succession of states will be repeated in time and the whole series of events will constitute a cycle. In general such a system is called a **clock**. Each period of the cycle then lasts **equally** long.

In particular note his reference to “...an absolutely isolated physical system (i.e. one not subject to external influences),” and compare it to Newton’s statement that, “Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external...” Clearly, Newton’s absolute time is unobservable – and from the standpoint of empirical science this is a serious fault which makes it unfit to serve as a standard of measurement. But in fact Weyl’s clock – defined as an absolutely isolated physical system – suffers from exactly the same problem: that which allows it

to be accurate makes it at the same time unobservable. Indeed, strictly speaking, Weyl's clock must be exactly the same thing as Newton's, since the only absolutely isolated physical system one can imagine is the universe itself.

The discovery of the finite velocity of light revealed that it is not possible to provide a univocal definition of time with respect to bodies of reference that are in relative motion. One consequence of this discovery, as Einstein observes, was to expose the previously unnoticed interdependence of space and time. That is to say, prior to the advent of relativity, points in space and instants of time were taken to be absolute realities – and time and space themselves were understood as completely different and independent things. In the special theory of relativity, Newton's absolutes are reorganized as follows:

1. We stipulate that the velocity of light is not only finite, but that it is an absolute constant – i.e. that its velocity in a vacuum is the same for all observers, independent of relative motion.
2. We stipulate that we will confine our attention to inertial reference frames – i.e. to bodies whose rate of relative motion is uniform.
3. And we continue to presuppose the absolutes of rigid transport and standard clocks.

Given these assumptions and qualifications, as Einstein observes, what has “physical reality” is neither points in space nor instants in time, but events, which are understood as specifications of four numbers in a space-time manifold. Accordingly, he observes,

There is no absolute (independent of the space of reference) relation in space, and no absolute relation in time between two events, but there is an absolute (independent of the space of reference) relation in space and time, as will appear in the sequel. (1922, p.30f.)

And in the following lecture he is equally clear about the fact that something like the Newtonian absolute is retained in the special theory of relativity (Einstein 1922, p.55):

The principle of inertia, in particular, seems to compel us to ascribe physically objective properties to the space-time continuum. Just as it was consistent from the Newtonian standpoint to make both the statements, *tempus est absolutum*, *spatium est absolutum*, so from the standpoint of the special theory of relativity we must say, *continuum spatii et temporis est absolutum*. In this latter statement *absolutum* means not only “physically real,” but also “independent in its physical properties, having a physical effect, but not itself influenced by physical conditions.”

But it is important that this paragraph occurs on the first page of the lecture in which Einstein is beginning to introduce the General Theory of Relativity – by contrasting it with the Special Theory. Because the point of this paragraph is that precisely that principle which seems to compel us to treat the space-time continuum as physically real and absolute – precisely that principle is what loses its privileged status under GR. If we attempt to generalize the theory of relativity to allow the description of the behavior

of bodies insofar as they are in mutually accelerated reference frames, we can no longer hope to describe this behavior accurately by means of coordinate systems referring to rigid reference bodies, nor can we assume that two clocks in different locations, both of which are at rest with respect to one reference body will give uniform readings when considered with respect to another reference body which is in non-uniform motion with respect to the first.

Our contention, therefore, is that it is only with the advent of GR that Einstein is forced to fully abandon the Newtonian absolute (understood in its 4-dimensional Minkowski version). But it is precisely at this point in the development of Einstein's theory that one discovers the need for what, in the following section, we call "the localized absolute."

That is to say, as soon as Einstein attempts to move beyond the restrictions of the special theory of relativity, two problems occur at the same time: One is that he is confronted by a need to give a consistent definition of a local clock. The other is that he needs to find or create something that can serve as an objective reference standard (which, as noted above, was one of the main intended – although not fulfilled – functions of Newton's notion of absolute time). But, paradoxical as it may seem, the abandonment of the rigidity of Minkowskian space-time means that the desired objective reference standard must be identified, not with some global frame of reference within which the local system is situated, but with the local system itself – insofar as the local system is taken as *the* system.

We resolve these two problems, *i.e.* the need for a properly defined local clock and an objective reference standard, by introducing the notion of the local time of a local system on the basis of a total universe, which is introduced as an objective reference standard. Our notion of local time, on the one hand, gives a consistent definition of local systems, each of which can accommodate a proper clock that serves the requirements of GR. We associate a local system to each classical point (t, x) of the 4 dimensional Riemannian manifold X , so that the local system can accommodate a local clock inside it which describes the local time at the center of mass of the local system.

On the other hand, we will show that it is possible to construct a concept of a "total universe" which will serve as the ground for an objective reference standard – without, however, violating the strictures of Einstein against an absolute reference frame in the classical Newtonian sense. We represent the total universe as an eigenstate of a Hamiltonian of infinite degrees of freedom. This allows to define the local time of each local system with finite degrees of freedom inside the total universe. We shall discuss this point in sections IV and V in detail.

The local system L at a classical point (t, x) in X has an internal structure which is *independent* of the classical mechanical world outside the local system L , as we will see in later sections. The internal structure of the local system L is described by quantum mechanics in our formulation. Since the local system L is independent of the external classical world, this assumption of our formulation does not lead to any contradiction as we shall see later. Furthermore, in the local system L a local time can be defined as a quantum mechanical notion associated with that local system L without requiring that time be defined as the problematic point-specific notion we encountered in Einstein's formulation of GR. This local time as defined above – specific to each local system – also

gives us a local time for the center of mass of L , thus satisfying the requirement of GR for a definition of local time valid at a particular point. Thus we can regard the centers of mass of various local systems as classical particles obeying GR, so that we can recover GR in this formulation of local systems and local times.

In this way the formulation outlined above illustrates the possibility of understanding QM and GR as mutually independent, but at the same time mutually complementary, such that each supports the other, supplying and supporting features that cannot be adequately defined within either theory by itself. QM provides the internal clock which recovers the local clock for GR, whose realization has been the intrinsic difficulty of GR from its very beginning as we have seen (see also section V below).

As a basis for this approach, we define, in the next section, the local clock of a local system as a measure of motion inside the local system. Our notion of local time will give us, as we explain below, a *localized* version of Newton's absolute time. As we shall see below, by "localizing" Newtonian time, we will be enabled to salvage that aspect of Newtonian time which is presupposed by quantum mechanics, without becoming entangled in conflict with the theory of relativity.

IV. Defining the Local Clock: an alternative notion of time

In this section we begin the presentation – to be continued in part II of this article – of an outline of our theory of local time, which we contend offers a possible way to unify Quantum Mechanics with General Relativity.

As noted in Kitada 1994a, p.283, the empirical scientist's notion of time *in actual practice* is necessarily a local one. That is to say, time does not appear to us until it is measured by some equipment. In this respect the observation of time is quite different from the observation of positions and motions, which are perceived directly by our senses. Even when we use a tool, such as a measuring stick, to measure the length of a thing, what we actually do is to look and see which markings on the scale of the ruler coincide with either extreme of the thing being measured. The fundamental act of observation here is the perception of this coincidence (of the ruler-mark with the edge of the thing). If we consider how time is measured by means of clocks, we notice that the measurement process is actually a process of comparing the motions and positions of certain bodies, so that it is possible to describe the measurement of time (somewhat abstractly) as a quotient of certain positions and velocities. With an analog clock, time is measured by examining the motion of its hands. We look at the hands, and recognize that one second passes if the second hand "moves" one "scale". We do not measure time directly by our senses, but we know time by perceiving the positions and motions of the hands of clocks. In this sense time is neither a quantity nor a frame given *a priori*. What exists first are the positions and movements of the bodies relative to our own position. The perception of the positions and motions indicates an introduction of the common parameter in each system of bodies consisting of a finite number of particles. This parameter is called time and it is a local notion by nature³.

³It should be noted here that the foregoing observations concerning the nature of time measurement

In QM, however, as we noted above in section II, the notion of time is quite different from that of Einstein. Obviously, as experimenters, quantum physicists use clocks in the same way that relativity physicists do, so that in practice they must implicitly share the understanding of time as a local notion. But *in theory* the Schrödinger equation has traditionally been understood to define the evolution of the particle states of a system with respect to an externally-given, “Newtonian,” time parameter, which means that for QM the clocks by which time is measured are understood as if they were completely external to the system being investigated – in sharp contrast to GR, where the fact that the clock is itself an object within the system is essential to the working of the theory. However, in recent years a group of mathematical physicists specializing in the study of Schrödinger wave operators in many-body systems, including Enss and Kitada, have made important strides in understanding the asymptotic completeness of observables in such systems. Building on the work of Enss, Kitada has shown that it is possible to re-interpret the role of the time parameter in the Schrödinger equation. Specifically, it is possible to treat the t in that equation, not as an observable, but as a dependent variable, whose specific meaning and value are derived, subject to appropriate assumptions, from a mathematical consideration based on the velocity and momentum of the particles of the system – which actually are observable. The mathematical reasoning which accomplishes this result is fairly technical – it will be summarized below and the reader is referred to Kitada 1994a, etc. for a more formal presentation of the argument. However, it is possible to state briefly here that this analysis, by inverting the customary relationship between the time parameter and the observation of the velocities and positions of particles as these things are interpreted by the Schrödinger equation, actually brings the understanding of time in QM into a much closer and more intelligible relation to that of GR than it had before. That is to say, the result of Kitada 1994a is to show that time may be understood in QM, no longer as an external “Newtonian” parameter, but as a certain expression of the relative motions and positions of the particles making up the local system. Thus we show that time may be understood as rooted in the internal motions of a local system in QM, just as it is in GR. At the same time, however, the theorem of Enss as interpreted and extended by Kitada 1994a allows us to explain why this “local time” must nonetheless appear as if it were an absolute, Newtonian time, independent of which point is selected within the local system and the same for all. And we will show later that it is precisely because it possesses this feature that the “local clock,” as we have defined it in the language of QM, provides the required logical foundation for the definition of local time in GR.

We are now in a position to define our notion of local time. We remark that the following exposition is a rather intuitive definition and precise formulation needs some mathematical notions and notations as described in sections 4 and 5, pp. 286-288 of Kitada 1994a. Let L be a local system consisting of N number of particles $1, 2, \dots, N$. Then there can be defined the position vectors x_1, x_2, \dots, x_N and momentum vectors $p_1 = m_1 v_1, p_2 = m_2 v_2, \dots, p_N = m_N v_N$, where m_j is the mass of the j -th particle, so that the correspondent quantum mechanical selfadjoint operators $X_j = (X_{j1}, X_{j2}, X_{j3})$ and

are in full agreement with Einstein, for it is clear that Einstein’s understanding of time is completely “operational,” as may be observed from the fact Einstein speaks primarily of clocks, not of time as a thing existing in itself.

$P_j = (P_{j1}, P_{j2}, P_{j3})$ in a Hilbert space $\mathcal{H} = L^2(R^{3n})$ of $N = (n + 1)$ particles satisfy the so-called canonical commutation relation. (This statement is axiom 2 of Kitada 1994a.) Then the local time t_L associated with the local system L is defined as a quotient of position x_j by velocity $v_j = p_j/m_j$

$$t_L = \frac{|x_j|}{|v_j|}. \quad (1)$$

Here we note that the right hand side of this definition looks as if it depends on the number j . But it is known (Enss 1986) that it does not depend on j , if one defines the right hand side as in Kitada 1994a, sections 4-5 (see axiom 3, Theorem 1, and Definitions 1-3 there, and section V of part II for more precise descriptions). Thus local time is defined as a *measure of motion* inside each local system.

We note that we have defined time only for local systems as a parameter of motions, which is abstracted from the internal motions inside the local systems. The fact that the universe as a whole is not a local system thus makes it reasonable to postulate that there is no time associated with the total universe. This postulate is axiom 1 of Kitada 1994a. This distinction between local systems and the total universe is seen more clearly when one notices that the local Hamiltonians describing local systems and the total Hamiltonian used to define the total universe differ in that the former is of finite degrees of freedom while the latter is of infinite degrees of freedom, and that the local Hamiltonians are no more than convenient approximations to the total Hamiltonian, used instead of the true, total Hamiltonian when one observes the outside. The fact that each of the local Hamiltonians of finite degrees of freedom is an approximation, but only an approximation, of the total Hamiltonian of infinite degrees of freedom has an interesting and important consequence: Namely it allows each local system to vary, so that local motions can occur and local clocks can be defined – even though the total universe, consisting of an infinite number of particles is stationary⁴, as we have postulated.

The stationary nature of the total universe will be described in section V of Part II in a more precise way. Any local system of a finite number of particles, however, can be nonstationary, and can vary inside itself, as a consequence of the variation outside the local system, which compensates for the change inside the local system, so that the stationary nature of the total universe is preserved.

Because of the fact that the relationship expressed in formula (1) has been shown to be independent of the particular choice of particle number, this quotient can be understood to hold approximately in the same way for any particle in the given local system – and precisely for this reason, this can be understood as defining a common parameter t_L associated with the local system L itself, rather than with any particular point in it, and which we can, therefore, define as the “local time” of the local system. Thus the demonstration, due to Enss, that this quotient holds independent of the choice of particle number j , is remarkable in that it not only gives us a way of understanding how a definition of time can be derived from the relative motions of particles within a local system, but at

⁴The word “stationary” here is the one used in mathematical physics to express an eigenstate of a Hamiltonian as a “stationary state.”

the same time it shows us that this parameter, because it is (approximately) independent of the particular choice of particle, can be treated as if it actually existed externally, independent of the motions on which it is based.

Nevertheless, once a local time is identified by the formula (1), as a measure of motion, as $t = |x|/|v|$ in each local system, our definition of local times is a specification or a clarification of the ‘relative, apparent, and common time’ measured ‘by the means of motion, which is used’ ‘instead of true time’ in Newton’s sense (see the first quotation from *Principia*, Newton 1962). It is a realization of Einstein’s local nature of time and coordinates, as well, in the sense that the local time is defined only for each local system consisting of a finite number of particles.

We note that there is a considerable difference between our definition of local times and the conventional understanding of the notion of time. The common feature of the conventional understanding of time, including Newton’s definition of absolute time, is that time is something existing or given *a priori*, independently of any of our activities, *e.g.* activities of observation. In our definition, time is not an *a priori* existence, but a convenient measure of motions inside each local system. Our definition of local times mentioned above is that a local time is a clock – which measures, not time, but the motions of the local system. Unlike the conventional understanding where time is given *a priori*, the local clock does not measure time, but it *is* time. Further, as we will state in section V, the proper clock is the local system itself, and it is a necessary manifestation of that local system. In this sense, “clocking” is the natural activity of any local system. It follows from this that to be an existing thing in the world necessarily involves clocking, without which there is no interaction. In these respects, our position is in complete opposition to the conventional understanding of time measurement, where time is given *a priori* and clocks measure those times, therefore the measurement of time is an incidental activity. Contrary to the conventional understanding, our view is that all beings are engaged in measuring and observing, and the activities of measuring and observing are not incidental, but pertain to the essence of all interactions. If we are permitted to express it somewhat boldly, we have turned things completely around: It is not that things exist and their duration is incidentally expressed by clocks. According to our formulation, clocks exist and their operation is necessarily expressed by duration.

Philosophically speaking, our understanding stated in axiom 1 about the totality of nature reflects that of Spinoza, especially insofar as Spinoza says that the totality of nature is Eternal, and defines Eternity as follows (Spinoza, Ethics, Part I, Definition 8, from E. Curley 1985 p.409):

D8: By eternity I understand existence itself, insofar as it is conceived to follow necessarily from the definition alone of the eternal thing.

Explanation: For such existence, like the essence of a thing, is conceived as an eternal truth, and on that account cannot be explained by duration or time, even if the duration is conceived to be without beginning or end.

Our axiom 1 which asserts that the total universe, which will be denoted ϕ , is stationary means in its mathematical formulation that it is an eigenstate of a total Hamiltonian H .

This means that the universe ϕ is an eternal truth, which cannot be explained in terms of duration or time. In fact, the eigenstate in itself contains no reference to time, as may be seen from its definition: $H\phi = \lambda\phi$ for some real number λ . The reader might think that this definition just states that the entire universe ϕ is frozen at an instant which lasts forever without a beginning or end. However, as we will see, the total universe ϕ has infinite degrees of freedom inside itself, as internal motion of finite and local systems, and never freezes. Therefore, as an existence itself, the universe ϕ does not change, however, at the same time, it is not frozen internally. These two seemingly contradictory aspects of the universe ϕ are possible by virtue of the quantum mechanical nature of the definition of eigenstates.

To sum up, the universe itself does not change. However, inside itself, the universe can vary quantum mechanically, in any local region or in any local system consisting of a finite number of (quantum mechanical) particles. Therefore, we can define a *local time* in each local system as a measure or a clock of (quantum mechanical) motions in that local system.

Let us consider, finally, the relationship that our notion of time bears to those of Newton and Einstein. First, we are in agreement with Einstein in abandoning the Newtonian conception of time as an absolute time pertaining to the entire universe. We have defined local times for describing local motions in a way that will be shown to be consistent with GR. However, we have also preserved certain aspects of the Newtonian conception: we have localized Newtonian time by showing that the local clock, as we have defined it, gives an approximation to Newtonian time which is valid for any particle of the local system. And, in another sense, we have retained for the universe as a whole the absoluteness of Newton – but without the flow of time – since our definition of local time involves the consequence (whose implications will be explored in Part II) that the universe itself is not in time, but is eternal, as Spinoza has defined that term.

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