

Comments on the Problem of Time

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Abstract. The problem of time, considered as a problem in the usual physical context, is reflected in relation with the paper by Kauffman and Smolin ([5]). It is shown that the problem is a misposed problem in the sense that it was raised with a lack of the recognition of mathematically known facts.

We found an idea on time similar to ours in the paper “A possible solution to the problem of time in quantum cosmology” ([5]) by Kauffman and Smolin.

Their argument concerning the nature of time seems to converge with our understanding of time as local notion ([1], [2], [3], [4]), even though we come at this issue from completely different angles.

That their idea might lead to our notion of local time can be seen especially from the summary passage in [5], near the end of the section entitled “Can we do physics without a constructible state space?”:

They offer a way of interpreting the evolution of quantum states in terms of a set of discrete, finite spin networks, each member of which has a successor network. They argue that by focusing attention on finite successor states it is possible to construct the relevant (local) probability amplitudes without constructing the total Hilbert space. They write:

The theory never has to ask about the whole space of states, it only explores a finite set of successor states at each step. ...

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The role of the space of all states is replaced by the notion of the successor states of a given network They are finite in number and constructible. They replace the idealization of all possible states that is used in ordinary quantum mechanics. ...

In such a formulation there is no need to construct the state space a priori, or equip it with a structure such as an inner product. One has simply a set of rules by which a set of possible configurations and histories of the universe is constructed by a finite procedure, given any initial state. In a sense it may be said that the system is constructing the space of its possible states and histories as it evolves.

Of course, were we to do this for all initial states, we would have constructed the entire state space of the theory. But there are an infinite number of possible initial states and, as we have been arguing, they may not be classifiable. In this case it is the evolution itself that constructs the subspace of the space of states that is needed to describe the possible futures of any given state. And by doing so the construction gives us an intrinsic notion of time.

In these passages, Kauffman and Smolin draw two particular conclusions about the nature of time, both of which correspond to the main points of our theory of local times: (a) that time is essentially local, and (b) that time is to be identified with the evolution of the system itself (which we call the “local system” in the following to indicate that it is a subsystem of the total system, and we call the time of the local system a local time of the local system.)

We now wish to make some comments on the proposal that Kauffman and Smolin offer as a possible solution for the problem of time (or, rather, the problem of the apparent absence of time). We will begin by arguing that the problem of time, as understood by Kauffman and Smolin and perhaps by physicists in general, involves a tacit assumption which is both fundamental and questionable. We will then show that this assumption is incorrect, and that, once it is rejected, a corrected formulation of the problem of time will allow for a different solution to the problem.

Kauffman and Smolin, and the other physicists whom they cite, appear to understand the problem of the absence of time as follows:

If the global time is absent in the sense that the constraint equation $Hf = 0$ holds for the total wave function f of the universe with H being the total Hamiltonian, then we have no time at every scale of the universe.

If the problem is stated in this way, and if we adopt their definition of local time as the local evolution itself, then the conclusion means that the local time defined as the evolution of a local system L cannot exist, and any state g of the local system L has to satisfy $H_L g = 0$. Here H_L is the local Hamiltonian of the local system L .

This conclusion — that the absence of global time implies the absence of local time — is the tacit assumption referred to above. It is not stated explicitly by Kauffman and Smolin. Indeed this implication does not seem to have been stated explicitly by anybody in the literature on this problem since it was first raised during the 1950's. But it is exactly this implication which is responsible for the feeling that we have a problem of the absence of time at *every* scale of the universe.

As a solution of this problem, they appeal to the conjecture that the total Hilbert space or the total wave function f “may not be constructible through any finite procedure” (Abstract of [5]). If this conjecture is true, then they can infer that the absence of time of the universe cannot be formulated in terms of any constructible procedure, therefore, the problem of time is “a pseudo-problem, because,” as they write, “the argument that time disappears from the theory depends on constructions that cannot be realized by any finite beings that live in the universe” (Abstract of [5]). Thus “the whole set up of the problem of time fails” (p.8, section 3 of [5]).

Their argument is thus summarized as follows.

There appears to be a problem of time because:

1. If we were to formulate the total wave function for the universe, we would find that time was absent at the global scale, and
2. The absence of global time implies the absence of local time.

But:

- 1a. The total wave function cannot be constructed by means of any finite procedure; hence,
- 2a. The absence of global time is never encountered as a problem by finite beings, therefore the implication of the absence of global time need never be faced by finite beings.

In other words, Kauffman and Smolin do not challenge the assumption that the absence of global time implies the absence of local time. Their implicit argument is that this assumption is not problematic, but just is inconsequential because its condition is never instantiated.

The following argument, however, shows that the assumption that the absence of global time implies the absence of local time is not inconsequential, but, on the contrary, has an implication which seriously undermines the conjecture of Kauffman and Smolin as to the nature of time.

Let B and S be two local systems such that B includes S as a subsystem. The same assumption which implies that the absence of global time implies the absence of local time would imply in this case that the time t_S of the local system S must be equal to the time t_B of the bigger system B .

If this is the implication of the thought which leads to the problem of time, there remains a problem in their solution, even if their conjecture is true. Consider two disjoint local systems S_1 and S_2 , and take a wave function g_j of the local system S_j ($j = 1, 2$). Let H_1 and H_2 denote the Hamiltonians associated with S_1 and S_2 . Then the time t_j of the system S_j is defined by the evolution on the state g_j . I.e., the local time of S_j is the t_j in the exponent of the evolution $\exp[-it_j H_j]g_j$ (for simplicity, we used this notation to denote the path integral which they used in their paper). Let S be the union of S_1 and S_2 . Then S is also a local system, and the tensor product $g = g_1 \otimes g_2$ is a wave function of the local system S . Moreover, the Hamiltonian H_S of S is given by $H_S = H_1 \otimes I + I \otimes H_2$, and the time t_S of the system S is given by the evolution $\exp[-it_S H_S]g$. According to the natural implication mentioned above, we then have that $t_j = t_S$ for $j = 1, 2$, because S includes both of S_1 and S_2 as subsystems of S .

Therefore, a natural extension of Kauffman and Smolin's argument yields that all local times of local systems must be identical with each other, and there is no local time which is compatible with the general theory of relativity.

Our suggestion is that to remedy this problem it is necessary to challenge the assumption that the absence of global time implies the absence of local time. As a sufficient basis for that challenge, we offer the following observation:

It is a known fact in mathematics, especially known in the area of mathematical scattering theory, that any local system (consisting of a finite number of QM particles) can have (internal) motion (i.e., can remain an unbound state with respect to the Hamiltonian associated to the local system), even if the total universe is a stationary (i.e. bound) state f in the sense that it satisfies the constraint equation: $Hf = 0$. Namely, this mathematical result means that the implication mentioned above:

$$Hf = 0 \text{ implies } H_L g = 0$$

does not necessarily hold in general.

To explain this result, we take a simple example of a local system consisting of 4 QM particles interacting by electronic Coulomb forces, one of which has positive charge, and the other 3 of which have negative charge. Then it is known (H. L. Cycon et al., "Schrödinger Operators," Springer-Verlag, 1987, p. 50) that this system has no eigenvalues. Namely, letting H_3 denote the Hamiltonian of this system, one has that every state $g \neq 0$ for this system does not satisfy the eigenequation $H_3 g = ag$ for any real number a .

However, one can construct a bound system by adding one particle with positive charge to this system so that one has a system consisting of 5 QM particles which has a bound state $f \neq 0$ for some eigenvalue b . Namely f satisfies $H_4 f = bf$ for a real number b , where H_4 denotes the Hamiltonian for the extended system of 5 particles. Thus we can regard this state f for this system of 5 particles a mini-universe, and the system consisting of 4 particles with Hamiltonian H_3 introduced at the beginning becomes a subsystem of the mini-universe. Exactly speaking, the state space of that subsystem of the mini-universe f should be the Hilbert space X_3 which includes all of the state functions $f(x, y)$ of the configuration x of the 4 particles, with the coordinate y (relative to the center of mass

of the first 4 particles) of the added 5th positive charged particle being arbitrary but fixed. Thus X_3 is a subspace of the Hilbert space of all the possible state functions for the Hamiltonian H_3 , and hence, by the above-mentioned result of the absence of eigenvalues for this Hamiltonian, we have that no nonzero state vector in X_3 is an eigenstate (i.e., bound state) of the Hamiltonian H_3 . Thus the subsystem with Hamiltonian H_3 of 4 particles is not a bound system. I.e., $H_3g = ag$ does not hold for any real number a and any vector $g \neq 0$ in X_3 .

This example shows that the usual supposition that a subsystem of a bound system is also a bound system is a mathematically incorrect statement. (This argument is a paraphrase of a paragraph of page 8 of [2], beginning with “To state this mathematically,”)

More exactly, we have the following theorem in the context of the simplest formulation of quantum mechanics:

Theorem. Let H be a N -body Hamiltonian with eigenprojection P (i.e., the orthogonal projection onto the space of all bound states of H), with suitable decay assumptions on the pair potentials. Let H be decomposed as follows:

$$H = H_1 + I_1 + T_1 = H_1 \otimes I + I_1 + I \otimes T_1,$$

where H_1 is a subsystem Hamiltonian with eigenprojection $P_1 = P_1 \otimes I$ (we use a simplified notation P_1 to denote the extension $P_1 \otimes I$, where \otimes denotes the tensor product operation and I is an identity operator), $I_1 = I_1(x, y)$ is the intercluster interaction among the clusters corresponding to the decomposition which yields the subsystem Hamiltonian H_1 , and T_1 is the intercluster free energy. Then we have

$$(1 - P_1)P \neq 0, \tag{1}$$

unless the interaction $I_1 = I_1(x, y)$ is a constant with respect to x for any y .

Remark. In the context of the former part of this paper, this theorem implies the following: Let L denote a sub local system of an N -body system with Hamiltonian H . Let H_L be the Hamiltonian of that local system and let P_L denote the eigenprojection for H_L . Then the above theorem yields the following:

$$(1 - P_L \otimes I)P\mathcal{H}_N \neq \{0\},$$

where \mathcal{H}_N is a Hilbert space of the N -body quantum system, which could be extended to the Hilbert space of the total universe in an appropriate sense (see [1] and [2]). This relation implies that there is some vector f in \mathcal{H}_N which satisfies that $Hf = bf$ for some real number b and that $H_Lg \neq ag$ for any real number a , where $g = f(\cdot, y)$ is a state vector of the subsystem L with an appropriate choice of the position vector y of the subsystem.

Proof of the theorem. Assume that (1) is incorrect. Then we have

$$(P_1 \otimes I)P = P.$$

Taking the adjoint operators on the both sides, we also have

$$P(P_1 \otimes I) = P.$$

Thus $[P_1 \otimes I, P] = (P_1 \otimes I)P - P(P_1 \otimes I) = 0$. But in generic this does not hold, because

$$[H_1, H] = \sum_j^{\text{finite sum}} c_j \frac{\partial}{\partial x_j} I_1(x, y) \quad (c_j \text{ being constants})$$

is not zero unless $I_1(x, y)$ is equal to a constant with respect to x . Q.E.D.

Our conclusion is that the absence of global time is compatible with the existence of local time, and the “problem of time” as stated by Kauffman and Smolin and other physicists is not a pseudoproblem, but an incorrectly formulated problem.

On the basis of this fact, we can construct the notion of local time as the evolution associated with each local system, which is proper to each local system and is compatible with the general theory of relativity, without contradicting the nonexistence of global time. See [2], and the references therein.

References

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